Crack Tunneling in Cement Sheath of Hydrocarbon Well

In a hydrocarbon well, cement fills the annular gap between two steel casings or between a steel casing and rock formation, forming a sheath that isolates fluids in different zones of the well. For a well as long as several kilometers, the cement sheath covers a large area and inevitably contains small cracks. The cement sheath fails when a small crack grows and tunnels through the length of the well. We calculate the energy release rate at a steady-state tunneling front as a function of the width of the tunnel. So long as the maximum energy release rate is below the fracture energy of the cement, tunnels of any width will not form. This failsafe condition requires no measurement of small cracks, but depends on material properties and loading conditions. We further show that the critical load for tunneling reduces significantly if the cement/casing and cement/formation interfaces slide. [DOI: 10.1115/1.4031649]

Keywords: wellbore cement, crack tunneling, friction

Introduction

Horizontal drilling and hydraulic fracturing have been pushing hydrocarbon wells to produce under extreme operational conditions and rigorous environmental regulations [1–4]. Geopolitical and financial turbulence aside, these technological advances have stimulated scientific research in oilfield mechanics and materials. Examples include mechanisms of hydraulic fracture [5–7], buckling in horizontal drilling [8], rheology of hydraulic fracturing proppants [9,10], mechanics of rubber–cement composite seals [11], swelling and leak of elastomeric seals [12–16], and downhole degradable composites [17,18].

This paper focuses on the fracture mechanics of cement, a material of choice in oilfields due to its low cost, low permeability, and ability to set in water, gas, and oil. Cement is commonly used as a sheath to fill the annular gap between two steel casings or between a steel casing and rock formation. The cement sheath prevents corrosion of the steel casings, provides mechanical support and, most importantly, isolates fluids in different zones of the well throughout its lifetime [19]. Even if the cement is initially properly set, it may damage due to, e.g., the drilling of wellbore, perforation of casing, and hydraulic stimulation of reservoir [20–22]. A damaged cement sheath may result in loss of hydrocarbon production, environmental pollution, and catastrophic disasters [23,24].

The current approach to the design of cement sheaths is through stress analysis [22,25–29]. This approach calculates the stress field using the linear theory of elasticity and obtains the critical condition for failure according to the Mohr–Coulomb or other strength criteria [22,25]. This approach is used to determine the critical external load and choose cement.

For a well as long as several kilometers, a cement sheath covers a large area and inevitably contains many small cracks, caused by, e.g., shrinkage during cement hydration, perforation, and hydraulic fracturing. So long as these small cracks remain stable, they do not impair the functions of the cement sheath. Under some conditions, however, the small cracks grow into large cracks. Growth of cracks in the radial direction of a cement sheath has been studied recently [30]. Such radial cracks cause local damage. The failure criterion developed in this approach depends on the location and size of the initial cracks, which cannot be measured in a long well.

Several experimental observations have led us to study a crack tunneling along the length of a cement sheath [29,31]. Boukhelifa et al. [32] observed that permeability of cement increased two orders of magnitude due to the crack generated by the loading cycles. We calculate the energy release rate at steady-state tunneling front. The energy release rate varies with the width of the tunnel and reaches the maximum value for a particular width of the tunnel. So long as the maximum energy release rate is below the fracture energy of the cement, tunnels of any width will not form. This condition provides a failsafe criterion that requires no measurement of the sizes of the small cracks, but depends on material properties and loading conditions. Furthermore, we show that the critical load for tunneling greatly reduces when frictional sliding of the cement/casing and cement/formation interfaces slides.

Steady-State Tunneling Crack

Consider a cement sheath, inner and outer radii $R_c$ and $R_b$, filling the gap between a steel casing and rock formation (Fig. 1(a)). Let $t_c = R_b - R_c$ be the thickness of the cement sheath and $t_r$ be the thickness of the steel casing. The pressure $p$ inside the steel casing may arise from changes in the downhole conditions or wellbore operations. The internal pressure causes a field of stress, taken to be invariant along the length of the well—that is, the well is assumed to deform under the plane strain conditions. In the absence of cracks, the hoop stress in the cement sheath is usually tensile, maximizes near the casing/cement interface, and decreases...
and may even become compressive as the cement/formation interface is approached [22].

A crack is confined within the cement sheath, but can tunnel along the length of the well. The crack propagating in cement is tortuous at a small scale, but creates a distinct path at a scale of the sheath, as observed in experiments [29,31]. In calculating energy release rate, we assume that the faces of the crack are flat. Let $l$ be the length and $h$ be the width of the tunnel. Also, the strength of cement is known to have pronounced tension–compression asymmetry. Here, we assume that the compressive strength is much larger than tensile strength and will only consider failure under tensile conditions. The energy release rate of the tunneling crack, $G$, is defined as the reduction of the potential energy of the system associated with the tunnel elongating along the length of the well by unit area. Everything else being fixed, the energy release rate of the tunneling crack increases with the length $l$. When $l/h > 2$, the tunneling crack approaches a steady state, namely, the energy release rate becomes independent of the length of the tunnel [33]. Following the linear elastic fracture mechanics, the normalized steady-state tunneling energy release rate takes the form

$$\frac{G_E}{R_b p^2} = M \left( \frac{h}{t_c} \right) \left( \frac{t_c}{t_B} \right) \left( \frac{E_c}{E_f} \right) \left( \frac{\nu_c}{\nu_f} \right)$$

where $M$ is a dimensionless function of dimensionless groups, $E$ and $v$ are the Young’s modulus and Poisson’s ratio, with the subscripts $c, f,$ and $s$ indicating the cement, formation, and steel casing, respectively.

The tunneling front advances in a three-dimensional elastic field. In the steady state, however, the energy release rate at the tunneling front can be calculated by using two fields under plane strain conditions. The first field exists far ahead the tunneling front, where $\sigma_0(r)$ is the hoop stress in the cement prior to the arrival of the tunneling front (Fig. 1(b)). The second field exists

![Fig. 1](image1.jpg) **Fig. 1** A crack tunnel in a cement sheath. (a) A cross section along the length of a well. A cement sheath fills the gap between a steel casing and rock formation. A crack tunnel in the cement sheath. (b) A cross section normal to the well, far ahead the tunnel front. The pressure inside the steel casing causes a stress field in the cement sheath. (c) A cross section normal to the well, far behind the tunnel front. The pressure inside the steel casing causes the crack to open in the cement.

![Fig. 2](image2.jpg) **Fig. 2** The energy release rate of a steady-state tunneling crack is a function of the width of the tunnel, $Q(h)$. (a) In some cases, the energy release rate reaches maximum $G_{\text{max}}$ when the width of the tunnel is less than the thickness of the cement sheath. (b) In other cases, the energy release rate monotonically increases with the width of the tunnel.

![Fig. 3](image3.jpg) **Fig. 3** The normalized energy release rate as a function of the normalized width of the tunnel: (a) $\nu_c=0.2$ and (b) $\nu_c=0.4$
far behind the tunneling front, where \( \delta(r) \) is the opening displacement of the crack (Fig. 1(c)). The energy release rate at the steady-state tunneling front is given by \[34\]

\[
G = \frac{1}{2h} \int_0^h \sigma_\theta(r) \delta(r) dr
\]

(2)

The calculations of \( \sigma_\theta(r) \) and \( \delta(r) \) are discussed in Appendices A and B, respectively.

The tunneling energy release rate is a function of the width of the tunnel, \( G(h) \). In some cases, the tunneling energy release rate reaches maximum when the width of the tunnel is less than the thickness of the cement sheath (Fig. 2(a)). In other cases, the tunneling energy release rate monotonically increases with the width of the tunnel (Fig. 2(b)). In either case, denote the maximum tunneling energy release rate by \( G_{\text{max}} \). Denote the fracture energy of the cement by \( \Gamma_c \). Small cracks of random sizes, orientations, and locations likely exist in the cement sheath. If \( G_{\text{max}} > \Gamma_c \), it is likely that some of the small cracks will grow into tunnels. If \( G_{\text{max}} < \Gamma_c \), however, no small cracks can grow into tunnels. Consequently, the condition \( G_{\text{max}} < \Gamma_c \) provides a failsafe criterion that requires no measurement of the small cracks.

**Failsafe Map**

The failsafe condition, however, depends on material properties and operating conditions. Consider a representative wellbore geometry, \( R_b/t_c = 5 \) and \( t_s/t_c = 0.5 \). The elastic constants of the steel casing and rock formation are taken to be constants:

\[
E_s = 200 \text{ GPa}, \quad \nu_s = 0.27, \quad E_f = 20 \text{ GPa}, \quad \text{and} \quad \nu_f = 0.2
\]

For the cement, two values of Poisson’s ratios are considered, \( \nu_c = 0.2 \) and 0.4, representing, respectively, regular cements and cement–rubber composites \[26,28,32\]. Young’s modulus of cement is chosen to be less than that of the rock formation, \( 0 < E_c/E_f < 1 \). We plot the normalized tunneling energy release rate \( G \) as a function of the normalized width of the tunnel (Fig. 3).

For the case \( E_c/E_f = 0.6 \) and \( \nu_c = 0.2 \), the tunneling energy release rate monotonically increases with \( h \) and reaches maximum when \( h = t_c \). For all the other three cases, energy release rates reach maximum when the width of the tunnel is less than the thickness of the cement sheath. The energy release rates can be dramatically decreased by decreasing Young’s modulus and increasing Poisson’s ratio of cement. Therefore, a more compliant and more incompressible cement can better resist tunneling cracks. This theoretical conclusion is consistent with field observations \[26,28,32\].

We represent the failsafe criterion in the plane with the modulus contrast \( E_c/E_f \) as the horizontal axis and the normalized fracture energy \( \Gamma_c/E_f/(R_bp^2) \) as the vertical axis (Fig. 4). The condition \( G_{\text{max}} = \Gamma_c \) divides the plane into two regions: tunneling and no tunneling. Tunnels will not form if the cement has a high-normalized fracture energy and low-normalized Young’s modulus. Also, a cement of a higher Poisson’s ratio is more tunnel-resistant. These failsafe maps can be used to determine the upper bound of the internal pressure \( p \) with the knowledge of cement properties or choose the proper cement based on the loading conditions. Approaches to avoid tunneling cracks include decreasing Young’s modulus of the cement, increasing Poisson’s ratio of the cement, or increasing the dimensionless group \( \Gamma_c/E_f/(R_bp^2) \). Cement with low Young’s modulus and high fracture energy is ideal to resist tunneling. However, the fracture energy often increases with the modulus for cement used in oilfields \[35\]. We consider correlations between fracture energy and Young’s modulus of three types: constant \( \Gamma_c \), linear dependence of \( \Gamma_c \) on \( E_c \), and quadratic dependence of \( \Gamma_c \) on \( E_c \) (Fig. 5). The three types assume \( \Gamma_c = 20 \text{ J/m}^2 \) at \( E_c/E_f = 0.6 \). Also included as dashed lines are the maximum energy release rates \( G_{\text{max}} \) for \( p = 30, 40, \) and 50 MPa. The failsafe condition \( G_{\text{max}} < \Gamma_c \) sets different requirements for cement, depending on the internal pressure and the type of \( \Gamma_c - E_c \) relation. For example, for the cement to avoid tunneling crack under internal pressure \( p = 40 \text{ MPa} \), the value of \( E_c/E_f \)
needs to be less than 0.5 for the type of constant $\Gamma_c$, 0.4 for the type of $\Gamma_c/C_{24}$, and 0.1 for the type of $\Gamma_c/C_{24}E_2$.

Effects of Interfacial Sliding

The above results assume that the cement/casing and cement/formation interfaces are well bonded. In oilfields, the interfaces may not be bonded, and frictional forces can be small, e.g., due to the contamination of oil-based drilling fluid [36]. The stress analysis assumes that the deformation is axisymmetric, so that interfacial sliding plays no role in predicting the failure of cement. By contrast, the magnitude of interfacial sliding can significantly affect the energy release rate of a radial crack [30]. Here, we examine the effect of interfacial sliding on tunneling cracks. The lower the friction forces, the larger the crack-opening displacement $\delta(r)$. Consequently, the energy release rate of tunneling crack will increase with the decreasing frictional forces.

First, we consider four extreme interfacial conditions, with each interface being either well-bonded, or capable of frictionless sliding (Fig. 6). No-sliding and free-sliding conditions for both interfaces provide a lower and upper bound, respectively. The maximum energy release rates increase about two orders of magnitude between the two bounds. The energy release rates for the four interfacial conditions for (b) $\nu_2 = 0.2$ and (c) $\nu_2 = 0.4$.

We also analyze cases with interfaces of finite friction coefficient $\mu$ (Fig. 7). In contrast to upper and lower bounds, the normalized energy release rates $G_{max}E_f/(R_{ref}^2)$ for finite friction coefficients are no longer constant with respect to $p$. They decrease with increasing $p$, because the frictional forces in the interfaces, which resist the crack from opening, become significant for large $p$. When $p = 50$ MPa, the case with $\mu = 0.01$ is close to free-sliding condition (upper bound) while the case with $\mu = 0.5$ almost converges to the no-sliding case (lower bound). The friction coefficients for the former and latter cases are typical values for cement/casing interface with and without contamination of drilling mud. These results further demonstrate the importance of mud-removal on the crack-resistance of cement.

Conclusions

This work studies cracks tunneling in cement sheath. Tunnels will not form if the maximum energy release rate is less than the fracture energy of the cement. This failsafe condition requires no measurement of small cracks, but depends on material properties and operating conditions. Cement with lower Young’s modulus and higher Poisson’s ratio better resists tunneling cracks. The decrease of Young’s modulus, however, often results in a decrease of fracture energy, which must be examined with care. Interfacial sliding can significantly reduce the critical load for tunneling crack.
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Appendix A: Stresses in Cement Sheath Prior to Cracking

The stress state prior to cracking can be obtained analytically as the solution to the Lame problem in the linear theory of elasticity. We assume that steel casing, cement, and formation are three coaxial cylinders with a different Young’s modulus and Poisson’s ratio. Under the axisymmetric condition, the strains in radial and circumferential directions, $e_r$ and $e_\theta$, are given by

$$ e_r = \frac{du}{dr}, \quad e_\theta = \frac{u}{r} \quad (A1) $$

where $u$ is the displacement in radial direction. The strains relate to the stresses by Hooke’s law

$$ e_r = \frac{1 - \nu^2}{E} (\sigma_r - \nu \sigma_\theta), \quad e_\theta = \frac{1 - \nu^2}{E} (\sigma_\theta - \nu \sigma_r) \quad (A2) $$

The balance of forces requires that

$$ \frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (A3) $$

Solving Eqs. (A1)–(A3)

$$ u_i = a_i r + \frac{b_i}{r} \quad \text{where} \ i = s, c, \text{and} \ f \quad (A4) $$

where $\sigma_r$ and $\sigma_\theta$ are the stresses in radial and circumferential directions.

Appendix B: Crack-Opening Displacement $\delta(r)$

We use a commercial finite-element software ABAQUS to simulate the crack opening inside the cement sheath. We simulate half of the cross section of the wellbore with a symmetric boundary conditions.

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Fig. 7 The effect of friction on the energy release rate. The modulus ratio of cement $E_c/E_f = 0.5$. Poisson’s ratios for the cement are chosen as: (a) $\nu_c = 0.2$ and (b) $\nu_c = 0.4$.

Fig. 8 The hoop stress, $\sigma_\theta$, in the cement sheath when $p = 50$ MPa, $E_f = 20$ GPa, $\nu_f = 0.2$, $R_c = 80$ mm, $t_s = 10$ mm, and $R_b = 100$ mm; (a) $\nu_c = 0.2$ and (b) $\nu_c = 0.4$.
profile, sufficiently refined meshes are applied in the vicinity of
ments are constructed (Fig.9((c))). To obtain accurate opening
result.

Eight-node biquadratic plane strain quadrilateral elements are
adopted for all the three layers of steel casing, cement sheath,
and formation except the crack tip region, where singular ele-
ments are constructed (Fig. 9(c)). To obtain accurate opening
profile, sufficiently refined meshes are applied in the vicinity of the crack tip.

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