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A FINITE ELEMENT METHOD FOR DIELECTRIC ELASTOMER TRANSDUCERS**

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ABSTRACT We present a finite element method for dielectric elastomer (DE) transducers based on the nonlinear field theory of DE. The method is implemented in the commercial finite element software ABAQUS, which provides a large library functions to describe finite elasticity. This method can be used to solve electromechanical coupling problems of DE transducers with complex configurations and under inhomogeneous deformation.

KEY WORDS finite element method, dielectric elastomer, electromechanical coupling

I. INTRODUCTION

A soft active material deforms in response to an environmental stimulus[1]. As a prominent example, when a membrane of a dielectric elastomer (DE) is subject to a voltage through its thickness, the membrane contracts in thickness and expands in area. About a decade ago it was discovered that voltage can cause a DE to strain beyond 100%[2]. This discovery has inspired an extensive development of DEs as sensors, actuators and generators—a broad-based technology known as artificial muscles[3–8]. In addition to large voltage-induced strains, desirable attributes of the DE transducers include light weight, rapid response, high energy density, silent operation, and low cost. The discovery has also motivated intense development of the theory of DEs[9–14].

The enthusiasm in the artificial-muscle technology also calls for the development of the computational mechanics of coupled large deformation and electrostatics. Dielectric elastomer transducers often integrate both soft and hard materials. Large deformation and electromechanical instability are keys to the understanding of the salient behavior of DEs[14]. Some of the envisioned applications, such as harvesting energy from ocean waves, involve fluid-structure interaction[15]. Coupled fields, however, are inadequately coded in existing commercial finite element method (FEM) software. For example, to work around the limitation of the existing software, several groups have implemented the electrical loading as an averaged Maxwell pressure[16,17], which makes the program unable to capture the localization and inhomogeneity of the electric field. A nonlinear field theory of dielectrics materials[11] has been used to study DE materials and actuators[18–23]. A user subroutine (UMAT) has been implemented in the commercial software ABAQUS, using the Gaussian-chain model (also known as the neo-Hookean model)[24].

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This UMAT is powerful to analyze DEs and systems. The Gaussian-chain model, however, is unsuitable for DEs when the stretch is large, approaching the limiting stretch of the polymer chains. Specifically, DEs often operate near the verge of the electromechanical instability, where the limiting stretch plays a prominent role[18]. Many hyperelastic models, such as the Ogden model[25], the Arruda-Boyce model[26], and the Gent model[27], are designed to capture the effect of the limiting stretch.

It is urgent to develop computational methods to analyze DE transducers under electromechanical loads, using hyperelastic models already existing in the commercial FEM software. Such methods will enable the use of the extensive capabilities of commercial software to analyze DE transducers of sophisticated maneuvers in a complex environment. The object of the present study is to develop such a method. The field equations of the DE membrane are described in §II. The method is described in detail in §III. The applicability of the method is illustrated through four examples in §IV.

II. A MODEL OF DIELECTRIC ELASTOMER MEMBRANE

The basic field equations of DE membranes are necessary for the following numerical applications. Here we adopt the same framework as in Ref.[24]. Consider a body undergoing an inhomogeneous deformation in three dimensions. Label each material particle by its coordinate $\boldsymbol{X}$ when the body is in the reference state. In a deformed state at time t, the material particle $\boldsymbol{X}$ moves to a new position of coordinate $\boldsymbol{x}$. The deformation of the body is described by the function

$$\boldsymbol{x} = \boldsymbol{x}\left(\boldsymbol{X}, t\right) \tag{1}$$

The deformation gradient $\boldsymbol{F}$ is defined as

$$F_{iK} = \frac{\partial x_i(\boldsymbol{X}, t)}{\partial X_K} \tag{2}$$

The body integrates a membrane of a DE, as well as passive materials typical in a DE transducer, such as a frame of a rigid material. The passive materials have already been treated in the existing FEM software. Here we focus on the membrane of DE. The membrane is sandwiched between two electrodes. In the reference state, when the membrane is undeformed, the thickness of the membrane can be inhomogeneous, and the shape of the membrane can be arbitrary. In the reference state, the thickness of the membrane is given as a function $\boldsymbol{H}\left(\boldsymbol{X}\right)$, and the uinivector normal to the membrane is given as the function $\boldsymbol{N}\left(\boldsymbol{X}\right)$. When a voltage Φ is applied between the two electrodes, the nominal electric field is

$$\tilde{\boldsymbol{E}} = \frac{\Phi}{\boldsymbol{H}}\boldsymbol{N} \tag{3}$$

The true electric field relates to the nominal one by

$$E_i = H_{ik}\tilde{E}_k \tag{4}$$

where $\boldsymbol{H}$ is the inverse of the deformation gradient $\boldsymbol{F}$.

The free energy density is a function of the deformation gradient $\boldsymbol{F}$ and the nominal electric field $\tilde{\boldsymbol{E}}$ is

$$\hat{W} = \hat{W}\left(\boldsymbol{F}, \tilde{\boldsymbol{E}}\right) \tag{5}$$

Specific free energy functions will be given later. Associated with small changes of the deformation gradient $\delta\boldsymbol{F}$ and the nominal electric field $\delta\tilde{\boldsymbol{E}}$, the free energy changes by

$$\delta\hat{W} = \frac{\partial\hat{W}\left(\boldsymbol{F}, \tilde{\boldsymbol{E}}\right)}{\partial F_{iK}}\delta F_{iK} + \frac{\partial\hat{W}\left(\boldsymbol{F}, \tilde{\boldsymbol{E}}\right)}{\partial \tilde{E}_J}\delta\tilde{E}_J \tag{6}$$

Equation (6) yields the nominal stress and the Cauchy stress as

$$s_{iK} = \frac{\partial\hat{W}\left(\boldsymbol{F}, \tilde{\boldsymbol{E}}\right)}{\partial F_{iK}} \tag{7a}$$

$$\sigma_{ij} = \frac{F_{jK}}{\det(\boldsymbol{F})} s_{iK} = \frac{F_{jK}}{\det(\boldsymbol{F})} \frac{\partial \hat{W}\left(\boldsymbol{F}, \tilde{\boldsymbol{E}}\right)}{\partial F_{iK}} \tag{7b}$$

In the current state at time t, $\boldsymbol{b}(\boldsymbol{X},t)\mathrm{d}V(\boldsymbol{X})$ represents the applied body force on a material element of volume, $\boldsymbol{t}(\boldsymbol{X},t)\mathrm{d}A(\boldsymbol{X})$ the applied traction on a materials element of surface, and $\rho(\boldsymbol{X})\mathrm{d}V(\boldsymbol{X})$ the mass of the material element of volume. Every material paricle of the system is in a state of local thermodynamic equilibrium if

$$\int \delta\hat{W}\mathrm{d}V = \int \left(b_i - \rho\frac{\partial^2 x_i}{\partial t^2}\right)\delta x_i \mathrm{d}V + \int t_i \delta x_i \mathrm{d}A \tag{8}$$

holds for arbitrary virtual displacement $\delta\boldsymbol{x}(\boldsymbol{X})$. The integrals extend over the whole volume and surface of the body, including both the DE membrane and the passive materials. Once Eqs.(2), (3) and (7a), (7b) are inserted, Eq.(8) defines the initial-value problem that evolves $\boldsymbol{x}(\boldsymbol{X},t)$. The applied voltage Φ and the applied mechanical forces constitute the loading parameters.

III. FEM IMPLEMENTATION

For problems with simple configurations and under homogeneous deformation, analytical solutions can be readily obtained. However, for practical design analysis in most cases, numerical methods should be developed. Well-developed commercial FEM software provides us powerful solvers. We adopt the commercial software ABAQUS, which has a broad user base, and an extensive library of hyperelastic models. We extend its capability by adding a user subroutine (UMAT) to model the DE membranes. The key process to embed the above method is to use the UMAT to define the constitutive behavior based on the free energy density function $\hat{W}$. Generally speaking, a unique UMAT has to be coded for a given $\hat{W}$, which makes the method inconvenient.

We specify the free energy function by using the model of ideal DEs[18]. This model has been widely used to analyze DEs, as reviewed in Ref.[14]. Within the model of ideal DEs, free energy density function $\hat{W}$ takes the form

$$\hat{W}\left(\boldsymbol{F}, \tilde{\boldsymbol{E}}\right) = W_{\mathrm{s}}(\boldsymbol{F}) - \frac{\varepsilon}{2}E_i E_i \tag{9}$$

$$\hat{W}\left(\boldsymbol{F}, \tilde{\boldsymbol{E}}\right) = W_{\mathrm{s}}(\boldsymbol{F}) - \frac{\varepsilon}{2\det\boldsymbol{F}} F_{iK}^{-1}\tilde{E}_K F_{iL}^{-1}\tilde{E}_L \tag{10}$$

The free energy of stretching, $W_{\mathrm{s}}(\boldsymbol{F})$, is taken to be any one of the well developed hyperelastic models, such as those of Ogden, Aruda-Boyce and Gent. The permittivity, ε , is taken to be a material constant independent of deformation and electric field.

Substituting the relation between the true stress $\boldsymbol{\sigma}$ and the nominal stress $\boldsymbol{s}$ into Eq.(7b) gives

$$\sigma_{im} = \frac{F_{mK}}{\det(\boldsymbol{F})}\frac{\partial W_{\mathrm{s}}(\boldsymbol{F})}{\partial F_{iK}} + \varepsilon H_{iK}H_{mL}\tilde{E}_K\tilde{E}_L - \frac{\varepsilon}{2}H_{jK}H_{jL}\tilde{E}_K\tilde{E}_L\delta_{im} \tag{11}$$

On the right side of Eq.(11), the first term comes from stretching the elastomer, and the other two terms come from the electric field. The Jacobian matrix of the constitutive model, $\partial\Delta\boldsymbol{\sigma}/\partial\Delta\boldsymbol{\varepsilon}$, can be obtained as a superposition of those from the stretching and the electric field, respectively. This fact, along with Eq.(11), paves the way for the development of the novel numerical method to study the DE transducers subjected to mechanical and electric loadings.

The fundamental idea here is that for a given set of finite element nodes, two sets of elements possessing the same nodes are defined such that each set of elements describe one special materials behavior, which is schematically demonstrated in Fig.1. This method is applicable to other problems when the true stress is the superposition of several terms, similar to Eq.(11). The additional required steps of the new method relative to the traditional FEM are generalized below:

(i) To select the proper form of $W_{\mathrm{s}}(\boldsymbol{F})$ available in the library of FEM software or well developed and apply it to one given set of elements;

(ii) To develop the constitutive relation with respect to each physical field and apply it to a new set of elements with the same nodes as those in step (i) one by one.

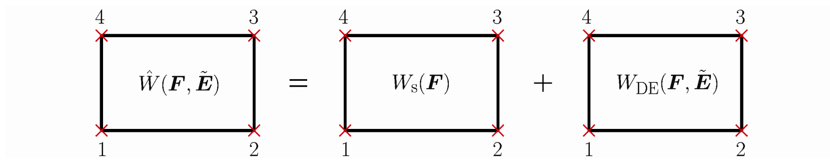

Fig. 1. Superposition of FEM elements: one related to stretching and the other related to polarization. $W_{\mathrm{s}}\left(\boldsymbol{F}\right)$ is the elastic deformation energy, and $W_{\mathrm{DE}}\left(\boldsymbol{F}, \tilde{\boldsymbol{E}}\right)$ is the electrical energy.

The most straightforward advantages of the above method can be clearly observed from Eq.(11): First, for a given material system, the material definition can be expanded by superposing contributions from different physical fields; Second, for given multiphysical fields, the material definition related to the stretching can make use of the rich library of the materials mechanical models in ABAQUS besides the superposition ability. Most important, the total number of nodal solution variables remains unchanged, which leads to the total computational cost not increasing much. However, special attentions must be paid when the computational results are processed. The elemental variables such as stress and energies of an integration point or a node should be added up for all of the elements with the same set of nodes, while the nodal variables do not need such summation. In ABAQUS, user subroutine UVAM can be used to generate output of the elemental variables. In the following section, verification and application of this numerical method are illustrated.

IV. NUMERICAL EXAMPLES

The method presented in §III is embedded in the commercial software ABAQUS for numerical study of materials and systems with complex configurations and under inhomogeneous deformations. Four examples of actuators made of DE are studied to verify the method.

The first term on the right-hand side of Eq.(10) is the elastic deformation energy function with several particular forms available in ABAQUS. Experimental observations have repeatedly shown that most materials are not described by the neo-Hookean model, while the Ogden model can reasonably capture the stretch-stiffening effect. For comparison, here we adopt both models. For the neo-Hookean model, the free energy takes the form

$$W_{\mathrm{s}} = \frac{\mu}{2}\left[\left(\det \boldsymbol{F}\right)^{-2/3} F_{iK}F_{iK} - 3\right] + \frac{K}{2}\left(\det \boldsymbol{F} - 1\right)^2 \tag{12}$$

For the Ogden model, the free energy takes the form

$$W_{\mathrm{s}} = \sum_{i=1}^{N} \frac{2\mu_i}{\alpha_i^2}\left(\bar{\lambda}_1^{\alpha_i} + \bar{\lambda}_2^{\alpha_i} + \bar{\lambda}_3^{\alpha_i} - 3\right) + \sum_{i=1}^{N} \frac{K_i}{2}\left(\det \boldsymbol{F} - 1\right)^{2i} \tag{13}$$

Here μ and μ_i are the initial shear modulus of Neo-Hookean and Ogden materials, respectively, while $K$and K_1 the bulk modulus. In Eq.(13), the value of N is specified by the user, those of μ_i, α_i are obtained through fitting experimental data. $\bar{\lambda}_i$ is defined as $\bar{\lambda}_i = \left(\det \boldsymbol{F}\right)^{-1/3}\lambda_i$ with λ_i representing the primary stretch. In the current study, the following values, which are obtained by fitting Fox et al.'s experimental data[28], are adopted for Ogden model: $N = 2$; $\mu_1 = 0.006\mu$, $\alpha_1 = 4.6829$; $\mu_2 = 0.994\mu$, $\alpha_2 = 0.9577$.

4.1. A Flat DE Membrane Subjected to a Voltage

For a flat DE membrane completely coated by electrodes on both faces as shown in Fig.2(a), the deformation in the elastomer is homogeneous until the nominal electric field reaches a maximum, where the electromechanical instability occurs. The analytical solution for this problem is

$$\tilde{E}\sqrt{\frac{\varepsilon}{\mu}} = \sqrt{-\lambda^3\frac{\mathrm{d}W_{\mathrm{s}}}{\mu \mathrm{d}\lambda}} \tag{14}$$

Figure 2(b) illustrates the comparison between the analytical and FEM results when the materials models take Neo-Hookean (Eq.(12)) and Ogden form (Eq.(13)), respectively. It is shown that the FEM

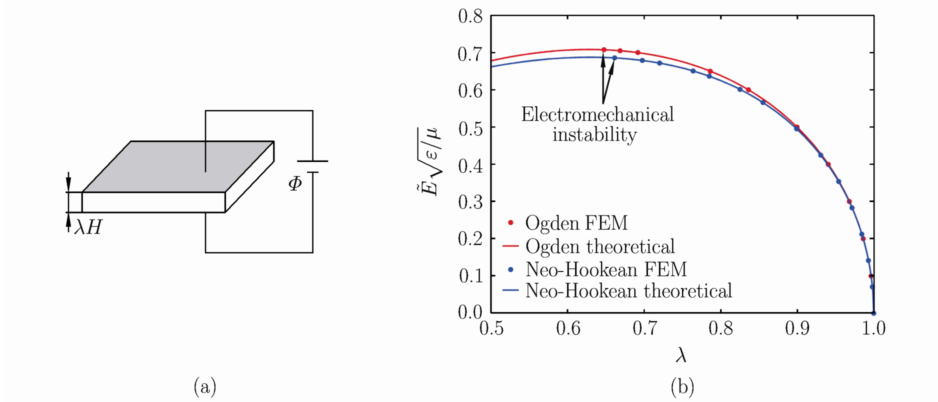

Fig. 2. (a) A flat DE membrane subjected to a voltage Φ; (b) FEM and theoretical results of normalized electric field *vs.* stretch for Neo-Hookean model and Ogden model.

results agree well with the analytical solution. Moreover, values of special variables with the occurrence of the electromechanical instability can be clearly obtained from numerical method rather than the analytical solution with further treatments.

4.2. A Balloon Subjected to Internal Pressure and a Voltage

A balloon with initial radius of R is subjected to internal pressure p and a voltage Φ as shown in Fig.3(a). Under a certain value of internal pressure, the relation between the nominal electric field and the stretch can be obtained as

$$\tilde{E}\sqrt{\frac{\varepsilon}{\mu}} = \sqrt{\frac{1}{2\lambda^3}\frac{\mathrm{d}W_\mathrm{s}}{\mu\mathrm{d}\lambda} - \frac{p}{\mu H/R}\frac{1}{2\lambda}} \tag{15}$$

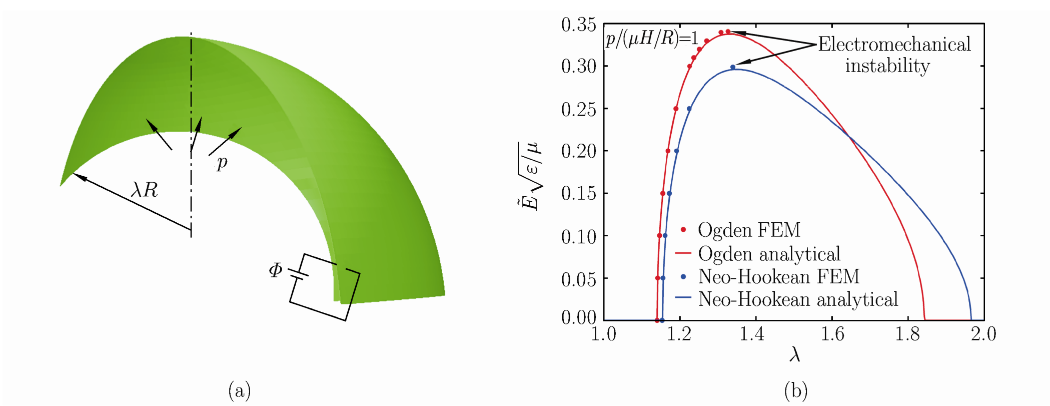

Fig. 3. (a) A balloon subjected to internal pressure p and a voltage Φ. (b) FEM and analytical results of normalized electric field *vs.* stretch for Neo-Hookean model and Ogden model.

Results obtained by Eq.(15) and the current method are shown as solid and dotted curves respectively in Fig.3(b) for both Neo-Hookean and Ogden models. It's observed that the current method agrees well with the analytical method.

4.3. A Circular Actuator Subjected to a Voltage

A circular layer partially coated by circular electrodes in the center of a large prestretched film are shown in Fig.4(a). The radii of the electrode coated and uncoated areas, i.e., the active and passive regions, are denoted as A and B, respectively. The prestretch value is defined as λ_{pre}.

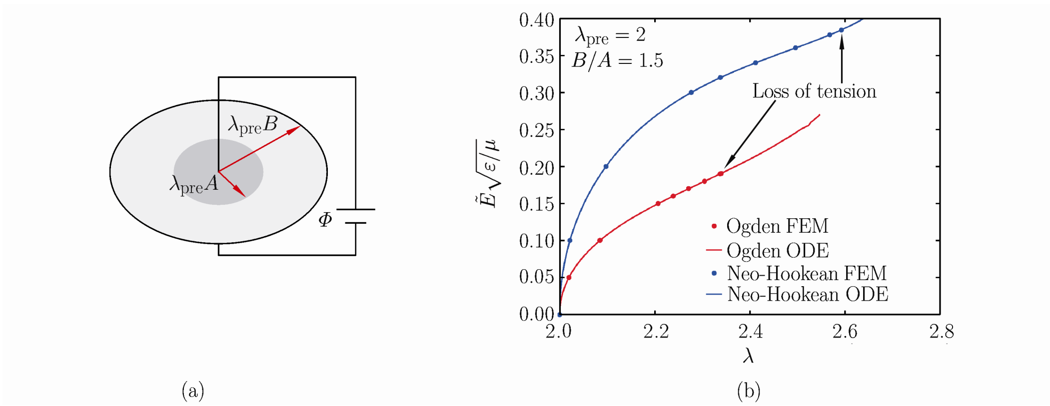

Fig. 4. (a) A circular actuator subjected to a voltage Φ; (b) FEM and analytical results of normalized electric field *vs.* stretch for Neo-Hookean model and Ogden model.

In the active region, deformation is a purely radial expansion with the total radial stretch λ_1 equal to the total circumferential stretch λ_2, i.e., $\lambda_1 = \lambda_2 = \lambda$. The incompressibility of the material therefore yields the third primary stretch as $\lambda_3 = \lambda^{-2}$. The nominal stress components are

$$s_1 = s_2 = s = \frac{1}{2}\left[\frac{\partial W_{\mathrm{s}}(\lambda)}{\partial \lambda} - 2\varepsilon \tilde{E}^2 \lambda^3\right] \tag{16}$$

In the passive region, the stresses and stretches are not uniform. The total radial stretch λ_1 and the total circumferential stretch λ_2 are

$$\lambda_1 = \frac{\partial r}{\partial R}, \qquad \lambda_2 = \frac{r}{R} \tag{17}$$

where r and R represent the radii of a material point in the intermediate and reference states, respectively. The third primary stretch is $\lambda_3 = \lambda_1^{-1}\lambda_2^{-1}$. The nominal stress components in the passive region are

$$s_1 = \frac{\partial W_{\mathrm{s}}(\lambda_1, \lambda_2)}{\partial \lambda_1}, \qquad s_2 = \frac{\partial W_{\mathrm{s}}(\lambda_1, \lambda_2)}{\partial \lambda_2} \tag{18}$$

The equilibrium equation is

$$\frac{\partial s_1}{\partial R} + \frac{s_1 - s_2}{R} = 0 \tag{19}$$

Combination of Eqs.(17)-(19) gives

$$\begin{aligned} \frac{\partial \lambda_1}{\partial R} &= \frac{\partial^2 W_{\mathrm{s}}(\lambda_1, \lambda_2)/\partial\lambda_1\partial\lambda_2\,(\lambda_1 - \lambda_2) + [\partial W_{\mathrm{s}}(\lambda_1, \lambda_2)/\partial\lambda_2 - \partial W_{\mathrm{s}}(\lambda_1, \lambda_2)/\partial\lambda_1]}{R\partial^2 W_{\mathrm{s}}(\lambda_1, \lambda_2)/\partial\lambda_1^2} \\ \frac{\partial \lambda_2}{\partial R} &= \frac{\lambda_1 - \lambda_2}{R} \end{aligned} \tag{20}$$

Equations (16) and (20) are the governing equations of the circular actuator model, and the following boundary conditions should be included to obtain the fields of stress and stretch in the film

$$s_1|_{R=A} = s|_{R=A}, \quad \lambda_2|_{R=A} = \lambda|_{R=A}, \quad \lambda_2|_{R=B} = \lambda_{\mathrm{pre}} \tag{21}$$

In the first two equations of (21), the variables in the left side belong to the passive region, while those in the right side belong to the active region. Substituting the stress and stretch in the active region into Eq.(16), the relation between the nominal electric field and the stretch for various prestretches can be obtained as

$$\tilde{E}\sqrt{\frac{\varepsilon}{\mu}} = \sqrt{\frac{1}{2\mu}\lambda^{-3}\frac{\partial W_{\mathrm{s}}}{\partial \lambda} - \frac{s}{\mu}\lambda^{-3}} \tag{22}$$

Results obtained by Eq.(22) with $B/A = 1.5$ and $\lambda_{\mathrm{pre}} = 2$ are shown in Fig.4(b) as solid curves, for Neo-Hookean and Ogden models, respectively. In the same figures, dotted lines are attained by the current method and they can reproduce the analytical results pretty well.

4.4. Frequency of DE Membrane Subjected to Pressure and a Voltage

The dynamics of a circular pre-stretched DE membrane subjected to a static pressure p and voltage Φ is studied. As shown in Fig.5(a), the DE membrane is first stretched radially with the prestretch values as λ_{pre}, then its boundary is fixed and a static pressure and a voltage are applied subsequently. Zhu et al. has performed the dynamic analysis of such a DE materials system[29].

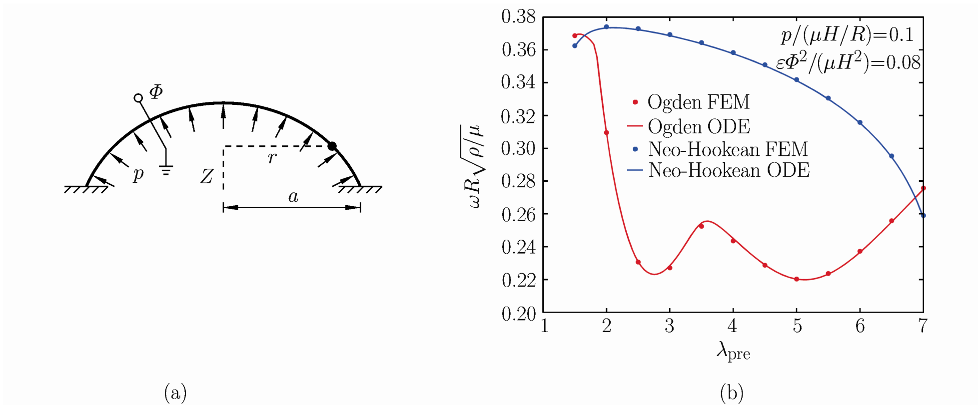

Fig. 5. (a) DE membrane subjected to pressure p and a voltage Φ; (b) FEM and analytical results of 1$^{\text{st}}$ dimensionless natural frequency *vs.* prestretch for Neo-Hookean model and Ogden model.

The relationship between the 1$^{\text{st}}$ dimensionless natural frequency and the prestretch is shown in Fig.5(b) with $pA/(\mu H) = 0.1$ and $\sqrt{\varepsilon/\mu}\Phi/H = 0.08$, here A is the initial radius of the membrane, and H the initial thickness. Both Neo-Hookean and Ogden materials models are used. Once again, the current method agrees well with the analytical method.

V. CONCLUSIONS

In summary, we have developed a finite element method for DE transducers based on the nonlinear field theory of dielectric materials. Within the model of DEs, the free energy is the sum of that due to stretching the network and that due to polarization. Similarly, the Cauchy stress also takes the form of superposition of two parts, and so is the Jacobian matrix. Therefore, for a given set of finite element nodes, two sets of elements are defined such that each kind of elements is adopted for either stretching or polarization. This method can be used widely to explore electromechanical coupling properties of DE transducers with complex configurations and under inhomogeneous deformation.

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