



How does a polymer glass resist fatigue crack growth?[†]

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We investigate fatigue crack growth in a polymer glass in which polymer chains are long and not crosslinked. Atoms bind by forces of two types: covalent bonds between repeat units along a chain, which resist chain scission, and noncovalent interactions between the chains, which resist chain slip. The covalent bonds are much stronger than the noncovalent interactions. When a crack impinges on a long chain, the chain slips and transmits tension over a segment of the chain. When the chain breaks at a single covalent bond, the energy stored in the segment dissipates. This molecular picture suggests a hypothesis: the fatigue threshold increases as the yield strength decreases. We analyze this hypothesis by developing a shear-lag model. We test the hypothesis by using high-molecular-weight poly(methyl methacrylate), and by modifying noncovalent interactions with plasticizers.

1. Introduction

Many applications are limited by fatigue crack growth in materials. When a material with a pre-cut crack is cyclically loaded, an amplitude of load exists, called the threshold, below which the crack does not grow. The threshold is a measure of the resistance of the material to fatigue. This paper focuses on the threshold of polymer glass.

Some polymers are rubbery at room temperature; they are often crosslinked into a network and are used as elastomers. Some polymers are glassy at room temperature; they are often not crosslinked and are used as thermoplastics. The threshold of elastomers has been extensively studied since the 1950s.^{1–5} In particular, Lake and Thomas noted that the threshold is around 50 J m^{-2} for various elastomers.⁴ They interpreted this finding in terms of a molecular picture (Fig. 1a). In an elastomer, the noncovalent interactions between chains are negligible compared to the covalent bonds that link repeat units of a chain and crosslink chains. The segment of a polymer chain between neighboring crosslinks is called a strand. When a crack impinges on a strand, the negligible noncovalent interactions allow the strand to slip readily so that the high tension transmits over the entire strand. That is, the high tension comparable to covalent strength is deconcentrated over the strand, rather than concentrated to an individual covalent bond along the backbone. When the strand breaks at a single covalent bond, the energy associated with the high tension in

the entire strand dissipates. This molecular picture of stress deconcentration estimates the threshold G_{th} by the covalent energy stored in a layer of strands per unit area:

$$G_{\text{th}} \sim eL_{\text{strand}} \quad (1)$$

where e is the energy of covalent bonds of backbone per unit volume, and L_{strand} is the end-to-end distance of the strand in the undeformed state. The longer the strand, the larger the amount of energy dissipated, and the higher the threshold.

The threshold is less understood for polymer glasses.⁵ The experimentally measured thresholds range between 10 to 100 J m^{-2} for polymer glasses, including poly(methyl methacrylate), polystyrene, and polycarbonate.^{6–9} The threshold of a polymer glass is higher than the threshold of an inorganic glass such as silica, $\sim 1 \text{ J m}^{-2}$.¹⁰ In a recent review on thresholds in polymers of various types, we have suggested a molecular picture of the threshold in a polymer glass (Fig. 1b).⁵ The noncovalent interactions between chains in a polymer glass are stronger than those in an elastomer, but are still much weaker than the covalent bonds along a chain. In a polymer glass of long chains, when a crack impinges on a chain, the chain does not break initially but slips so that the high tension still transmits over a long segment of the chain. Unlike an elastomer network, the polymer glass is not crosslinked, so the segment of high tension is not confined by crosslinks. Neither is it confined by neighboring entanglements because the entanglements slip. Because the chain is long in the polymer glass, the chain slip is jammed by the noncovalent interactions between the chains (*i.e.*, interchain friction). This jamming is modeled by the shear lag model, which estimates the length of the segment of high tension. When the chain breaks at a single covalent bond, the energy stored in the segment dissipates.

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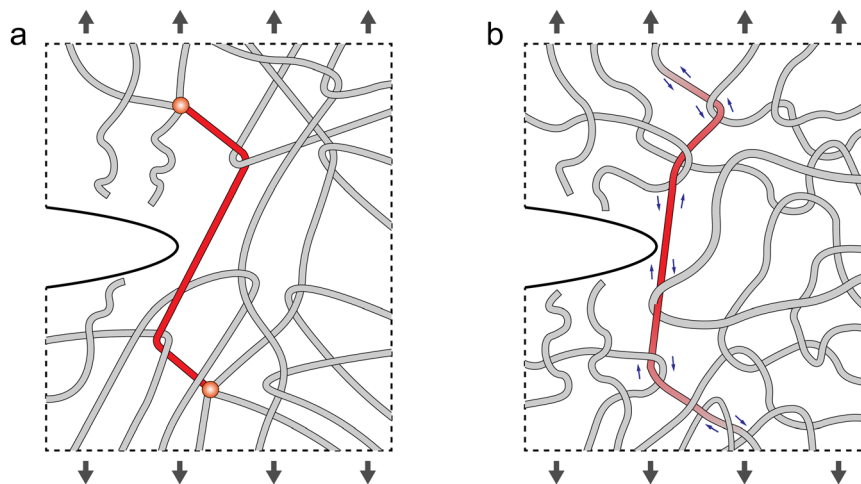


Fig. 1 Comparison of a crack in an elastomer and a crack in a polymer glass. (a) In an elastomer, chains are crosslinked. When a crack impinges on a chain in the elastomer, the entire chain approaches the covalent bond strength. (b) In a polymer glass, chains are uncrosslinked. When a crack impinges on a chain in the polymer glass, a segment of the chain approaches the covalent bond strength. The length of the segment of high tension is set by neither crosslinks nor entanglements, but by the jamming of the chain through noncovalent interactions with other chains.

The weaker the noncovalent interactions between the chains, the longer the segment over which the high tension transmits, and the larger the threshold.

In a polymer glass, the noncovalent interactions between chains set the yield strength. The above molecular picture suggests a hypothesis: as the yield strength decreases, the threshold increases. Here, we test this hypothesis by preparing two families of polymer glasses and separately measuring their yield strengths and fatigue thresholds. Our experiments show that the threshold increases as the yield strength decreases. As a crack grows in a polymer glass, both chain slip and chain scission occur. To ensure the crack growth by chain scission, we prepare poly(methyl methacrylate) (PMMA) of long chains through free radical polymerization using an extremely low fraction of photoinitiator. We vary noncovalent interactions between chains using various mass fractions of plasticizers of two kinds: dioctyl phthalate (DOP) and 1-butyl-3-methylimidazolium hexafluorophosphate (BMIM-PF₆). When the yield strength decreases from ~ 45 MPa to ~ 10 MPa, the fatigue threshold increases from ~ 60 J m⁻² to ~ 220 J m⁻².

2. Shear lag model for the threshold of a polymer glass

This molecular picture has been analyzed by a shear lag model.⁵ This model is summarized here to provide a background. The Lake-Thomas model does not apply to polymer glasses for two reasons: the chains have significant noncovalent interactions, and the chains are uncrosslinked. The concept of stress deconcentration, however, can still apply to polymer glasses. When a crack impinges on a chain in a polymer glass of high molecular weight, the chain slips over a segment and is jammed by interchain friction. After the slip is jammed, the crack grows by chain scission. Before chain scission, chain slip

allows the high tension to transmit over a segment of the chain. When the chain breaks at a single covalent bond, the energy associated with high tension in the segment dissipates. The threshold G_{th} scales as

$$G_{\text{th}} \sim eL \quad (2)$$

where e is the energy of covalent bonds of backbone per unit volume, and L is the end-to-end distance of the polymer segment over which high tension is transmitted. Note that L is calculated in the undeformed state.

Consider a polymer glass with a pre-cut crack subject to a load (Fig. 2a). The noncovalent interactions between chains are much weaker than the covalent bonds along the chains. Consequently, when the crack impinges on a chain, the chain does not break initially but slips. In a polymer glass of high molecular weight, at the deformed state, the high tension can transmit over a segment of length l , which is much larger than the length of a covalent bond (Fig. 2b). We estimate l by applying the shear lag model to the chain.

In the deformed state, at the crack tip, the tensile stress along the chain, S_{chain} , is limited by the strength of a covalent bond. Away from the tip, the stress S_{far} is much lower than S_{chain} . Denote the interchain frictional stress by $\tau_{\text{interchain}}$ and diameter of the polymer chain by b (Fig. 2c). The balance of forces acting on the chain requires that

$$\tau_{\text{interchain}}\pi b l/2 = S_{\text{chain}}\pi(b/2)^2 \quad (3)$$

Solving l and neglecting a numerical prefactor, we obtain that

$$l \sim (S_{\text{chain}}/\tau_{\text{interchain}})b \quad (4)$$

Thus, the number of repeat units along the segment of the chain scales by

$$n \sim (S_{\text{chain}}/\tau_{\text{interchain}}) \quad (5)$$

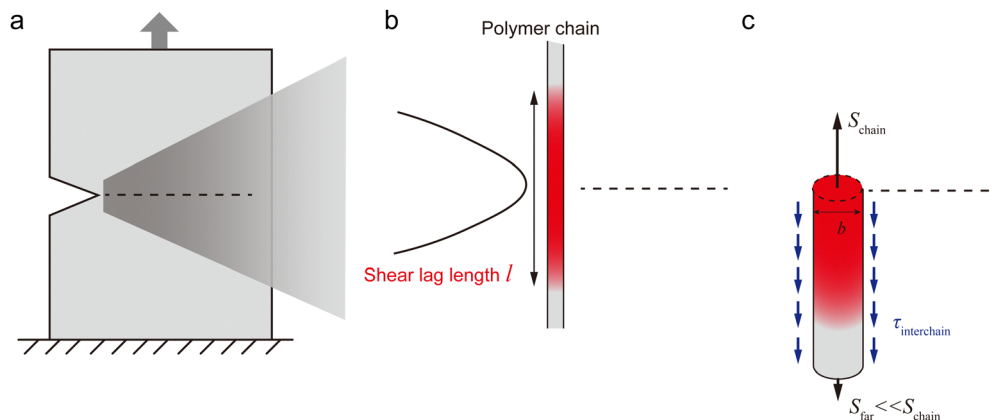


Fig. 2 Shear lag model of a polymer glass. (a) A polymer glass with a pre-cut crack is subject to a load. (b) When the crack impinges upon a chain, the chain slips and transmits high tension over a segment of length l . (c) Free body diagram of the segment of the polymer chain.

The straightened polymer segment in the undeformed state is coiled, taken to be in the shape of a random walk. Consequently, the end-to-end length of the segment in the undeformed state is

$$L = n^{1/2}b \sim (S_{\text{chain}}/\tau_{\text{interchain}})^{1/2}b \quad (6)$$

A combination of eqn (2) and (6) estimates the threshold of a polymer glass by

$$G_{\text{th}} \sim e(S_{\text{chain}}/\tau_{\text{interchain}})^{1/2}b \quad (7)$$

Taking $S_{\text{chain}} = 10$ GPa, $\tau_{\text{interchain}} = 10$ MPa, $b = 5 \times 10^{-10}$ m, the shear lag length is $l = 5 \times 10^{-7}$ m, which corresponds to $n = 1000$ repeat units. In this context, a polymer glass is considered to have high molecular weight if the molecular weight is large compared to that set by the shear lag length. Take the energy of a covalent bond $J = 3.3 \times 10^{-19}$ J and the volume of a covalent bond $v = 4 \times 10^{-29}$ m³, so that $e = J/v = 8.25 \times 10^9$ J m⁻³. Note that the value of e would be lower than estimated here since the covalent bonds in the side groups do not bear tension, and therefore do not contribute to the fatigue threshold. This effect is neglected in scaling relations (eqn (1) and (2)). The shear lag model predicts the threshold of a polymer glass to be 42 J m⁻². This estimate falls in the range of the experimentally measured threshold for various polymer glasses.^{6–9}

The interchain friction scales with the yield strength of the polymer glass, $\tau_{\text{interchain}} \sim s_Y$. Consequently, eqn (7) suggests a scaling relation between the threshold and yield strength:

$$G_{\text{th}} \sim s_Y^{-1/2} \quad (8)$$

Polymer glasses often have chains of over 10 000 repeat units, but shear lag limits stress deconcentration to 1000 repeat units. Increasing interchain friction reduces the length over which high tension transmits.

In the limit of high interchain friction ($\tau_{\text{interchain}} \rightarrow S_{\text{chain}}$), the polymer glass behaves like an inorganic glass such as silica, so that

$$G_{\text{th}} \sim eb \approx 4 \text{ J m}^{-2} \quad (9)$$

This prediction is on the same order of magnitude as the threshold measured experimentally.^{6–9}

Note here that when we describe both the Lake-Thomas model and the shear lag model, we only focus on the scaling relation and ignore some constant and other molecular details. For example, in an elastomer, as the crack grows, it may dissipate energy in several layers of strands. Moreover, one should consider the effect of the components that do not bear loads, such as side groups and liquid additives. When the polymer chain is far from a freely-jointed chain, one should consider using Kuhn length and number of Kuhn segments to calculate the end-to-end distance.

3. Results and discussion

We hypothesize that, for polymer glasses of long chains, the fatigue threshold increases as the yield strength decreases. To test this hypothesis, we prepare polymer glasses of various yield strengths and measure the fatigue thresholds of these materials. To have a family of polymer glasses in which cracks grow by chain scission, rather than chain slip, we prepare PMMA of long chains. To vary yield strength, we add various mass fractions of a plasticizer to PMMA. The plasticizer used in this study is either the 1-butyl-3-methylimidazolium hexafluorophosphate (BMIM-PF₆) or dioctyl phthalate (DOP).

We measure the stress–strain curves of the two families of polymer glasses, PMMA/BMIM-PF₆ and PMMA/DOP (Fig. 3a and c). For the pure PMMA and PMMA with low mass fractions of plasticizer, the samples gradually whiten under stress and suffer brittle rupture at small strains (Movie S1, ESI†). The stress-whitening is ascribed to the formation of craze of the polymer glass.⁸ During crazing, microvoids and fibrils form. The microvoids further develop into cavities and microcracks, which scatter light. When the fibrils break, a sample fractures.

At intermediate mass fractions of plasticizer, the samples are ductile with large plastic deformation. At the initial stage of loading, a localized, tilted whitening region appears, and then the whitening region further develops into a neck. The strain of the neck is stabilized, possibly because the entanglements are

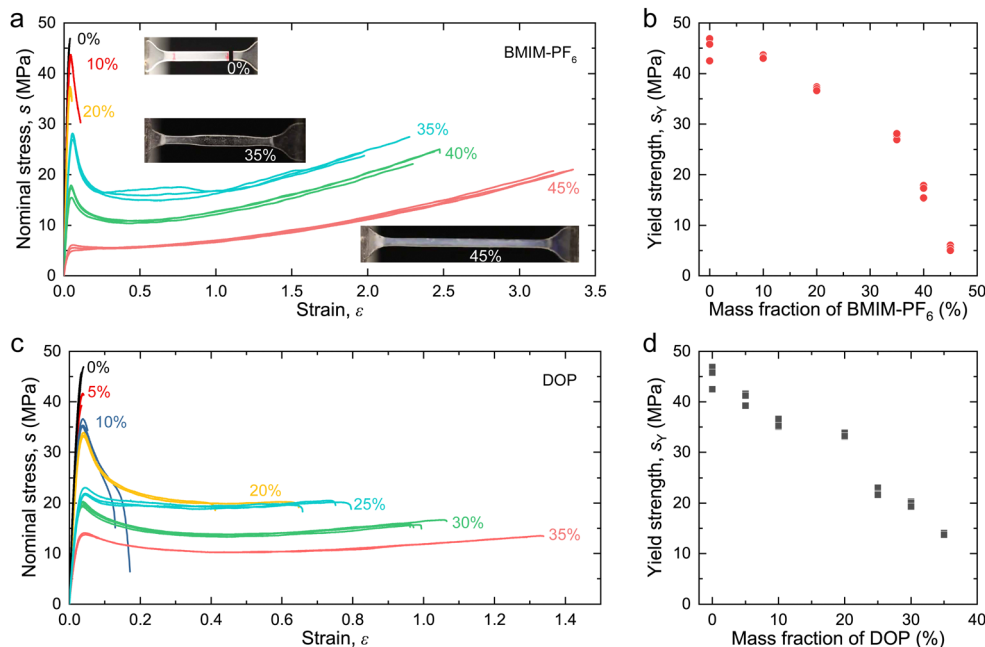


Fig. 3 Effect of the plasticizers on the stress–strain curves. (a) Stress–strain curves of the samples with various mass fractions of BMIM-PF₆. (b) Yield strength as a function of mass fraction of BMIM-PF₆. (c) Stress–strain curves of the samples with various mass fractions of DOP. (d) Yield strength as a function of mass fraction of DOP.

jammed and prevent the chains from further slipping. The neck of nearly constant strain then elongates to the whole sample. After the entire length of the sample is necked, the chain slip remains to be jammed, leading to chain scission, resulting in ductile fracture (Movie S2, ESI†).

At high mass fractions of plasticizer, the samples gradually whiten and undergo a homogeneous deformation until break (Movie S3, ESI†). The interchain friction is so low that the chains slip readily, resulting in ductile fracture.

For the pure PMMA, we take the stress at break as the yield strength. For other samples, we take the initial peak stress as the yield strength. The plasticizers decrease interchain friction. As the mass fraction of plasticizer increases, the yield strength decreases from ~ 45 MPa to ~ 10 MPa (Fig. 3b and d). For PMMA plasticized by DOP, we can make homogenous samples up to a mass fraction of 35% of DOP. Above this mass fraction, the PMMA-DOP mixture separates into two phases. For PMMA plasticized by BMIM-PF₆, we fabricate samples with a mass fraction of BMIM-PF₆, up to 45%. Beyond this mass fraction, the crack grows at low amplitudes of load, possibly by chain slip.

We measure fatigue crack growth in these two families of polymer glasses. We precut the samples and stretch the samples at an amplitude of stress. The amplitude of energy release rate is calculated by the amplitude of stress, with details referred to the Experimental section. We plot the crack growth per cycle, dc/dn , as a function of the amplitude of the energy release rate, G (Fig. 4). A value of the amplitude of energy release rate exists, G_{th} , below which the crack does not grow. The fatigue threshold G_{th} increases with the mass fraction of plasticizer (Fig. 4). The fatigue threshold is ~ 60 J m⁻² for pure PMMA, ~ 160 J m⁻² for PMMA with 30% of DOP (Fig. 4a), and ~ 250 J m⁻² PMMA with 40%

BMIM-PF₆ (Fig. 4b). Note that for the pure PMMA, the present value of fatigue threshold is comparable to that reported in literature.^{7,9}

We observe the crack tip of the sample of various mass fractions of the plasticizers using a microscope. When the mass fraction of BMIM-PF₆ is low to moderate ($< 40\%$), the crack tips are sharp (Fig. 5a). The crack grows by a finite length per cycle during the cyclic loading. When the mass fraction of BMIM-PF₆ is 45%, the crack blunts (Fig. 5b). The crack grows even when the amplitude of energy release rate is 35 J m⁻². That is, the excessive mass fraction of plasticizer causes chains to slip readily so that the crack may grow by chain slip without chain scission.

We plot the fatigue threshold as a function of yield strength (Fig. 6). Despite the different chemical structures of the two plasticizers, DOP and BMIM-PF₆, the fatigue threshold of the two families of polymer glasses increases as the yield strength decreases. These data are consistent with our hypothesis that polymer glass resists fatigue crack growth by deconcentrating stress at the crack tip. For polymer glass of long chains, the lower the interchain friction, the longer the segment of chain over which the high tension transmits. The interchain friction sets the yield strength, and the length scale of stress deconcentration sets the fatigue threshold. The length scale of stress deconcentration can be understood by the shear lag model as described in Section 2. This result implies a threshold-strength conflict: the fatigue threshold increases as the yield strength decreases.

It should be noted here that the experimental results do not exactly follow the scaling law of the shear lag model (eqn (8)). The possible reason might be that the model is simplified.

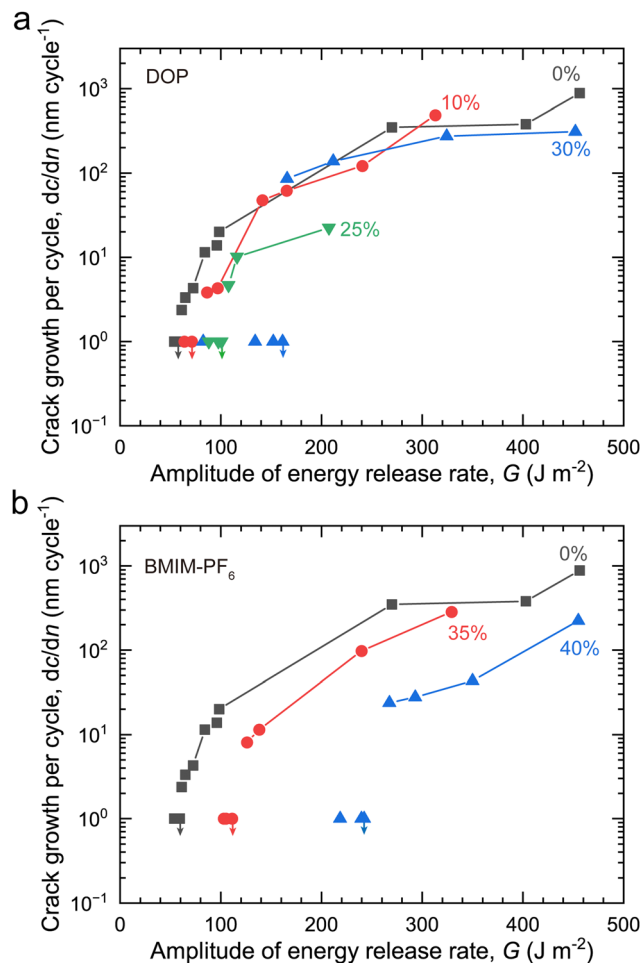


Fig. 4 Crack growth per cycle as a function of the amplitude of energy release rate. (a) PMMA with various mass fractions of DOP; (b) PMMA with various mass fractions of BMIM-PF₆.

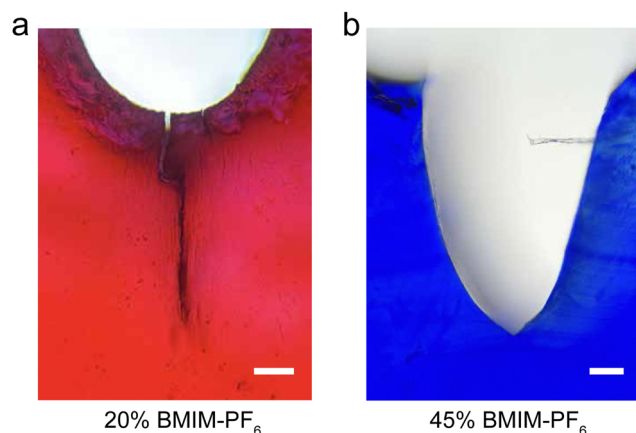


Fig. 5 Optical microscopy images at the crack tip of (a) PMMA with 20% of BMIM-PF₆ (b) PMMA with 45% of BMIM-PF₆. The scale bars represent 100 μm .

For instance, cyclic load may cause the polymer chain to slip over a longer distance by plastic ratcheting, a phenomenon

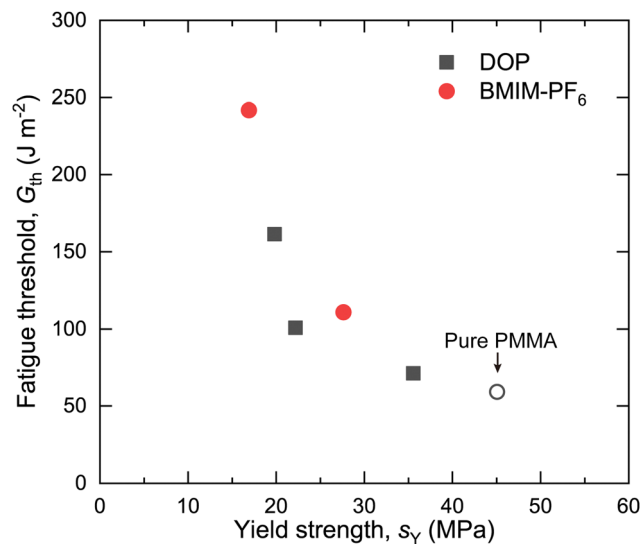


Fig. 6 Fatigue threshold as a function of yield strength.

analogous to that observed in a textile.^{11,12} This effect may lead to a larger length scale of stress deconcentration. The shear lag model presented in Section 2 does not capture plastic ratcheting. Besides, in reality, the polymer segment is curved so that the shear force is not in a single direction. Here, we use the simplified model to represent stress deconcentration. Quantitative comparison requires a more detailed model than the shear lag model.

Note here that in this scaling analysis, we do not take the polymer segment between the neighboring entanglement as a length scale. When a crack impinges on a chain, the tension in the chain is limited by covalent bonds, which are much stronger than the noncovalent interactions between the chain and neighboring chains. Entanglements slip so that high tension in the chains readily transmits beyond entanglements. This hypothesis has been tested in entangled hydrogels and elastomers.¹³ We should note that entanglements do play significant roles in the mechanical behavior of glassy polymers, such as stabilizing necks and crazes.^{14–19} Such effects of entanglements are not studied in this paper.

4. Conclusion

We hypothesize that, for polymer glasses of long chains, the fatigue threshold increases as the yield strength decreases. We motivate this hypothesis using a shear lag model. Atoms in a polymer glass bind in two ways: covalent bonds between repeat units along a chain, which resist chain scission, and noncovalent interactions between the chains, which resist chain slip. The covalent bonds are much stronger than the noncovalent interactions. When a crack impinges on a chain, the chain slips and transmits high tension along the chain. Because the chain is long, interchain friction jams the chain slip so that high tension transmits over a segment of the chain. When the chain breaks at a single covalent bond, the energy stored in the segment dissipates. The weaker the noncovalent interactions between the chains, the longer the segment over which the high

tension transmits, and the higher the fatigue threshold. To test this hypothesis, we prepare PMMA of long chains and with two plasticizers of different chemical structures. When the yield strength decreases from ~ 45 MPa to ~ 10 MPa, the fatigue threshold increases from ~ 60 J m $^{-2}$ to ~ 220 J m $^{-2}$. These results identify a conflict between fatigue threshold and yield strength. Load-bearing applications of polymer glasses commonly require both high fatigue threshold and high yield strength. The former enhances durability, and the latter resists permanent deformation. Resolving this threshold-strength conflict will extend the scope of applications of polymer glasses.

5. Experimental section

5.1. Sample preparation

We prepare poly(methyl methacrylate) (PMMA) with various mass fractions of plasticizers as follows. All chemicals were purchased and used without any purification. A PMMA precursor is prepared by mixing methyl methacrylate (MMA, Sigma Aldrich, M55909) monomer and photoinitiator 2-hydroxy-4'-(2-hydroxyethoxy)-2-methylpropiophenone (Irgacure 2959, Sigma Aldrich, 410896). The initiator-to-monomer molar ratio is 10^{-5} . We use plasticizers of two kinds: dioctyl phthalate (DOP, Sigma Aldrich, D201154) and an ionic liquid (1-butyl-3-methylimidazolium hexafluorophosphate, Sigma Aldrich, 70956). We add a mass fraction of a plasticizer in the PMMA precursor. The precursor is then poured into a mold made of a glass plate and a silicone spacer (McMaster, 1460N11). The mold is covered with another glass plate, and the two glass plates are clipped using binder clips. The sample clipped between the glass plates is then placed in a polyethylene bag (VWR, 4662002). The mixture is polymerized under ultraviolet irradiation (15 W 265 nm; UVP XX-15L) for 24 hours. The prepared PMMA sheet has a thickness of ~ 0.8 mm. The sample is taken out of the polyethylene bag and placed in a hood for 12 hours to evaporate unreacted monomers.

5.2. Uniaxial tensile tests

We use a laser cutter (Helix 75 W, Epilog Laser) to cut a dogbone-shaped specimen with gauge length of 12 mm and width of 2 mm, following the ISO 527-2-5B standard. We mark two points at the end of the gauge length of the specimen. The specimen is pulled by a tensile tester (Instron; 5966) with a 10 kN load cell, at a strain rate of 0.01 s $^{-1}$ until break. While the force is recorded, the deformation is videotaped. Nominal stress is defined as the force divided by the cross-sectional area of the undeformed specimen. Strain is defined as the displacement between the two marked points divided by the undeformed gauge length. For PMMA with each mass fraction of a plasticizer, three specimens are tested.

5.3. Fatigue tests

The fatigue test is performed by using a sample with a single-edge crack. The sample is cut using the laser cutter into a rectangular shape with width of 20 mm and length of 45 mm. A precut crack of length ~ 6 mm is introduced by using the laser

cutter. To make a sharp crack in the beginning, we let the precut crack grow a little by pulling the specimen by a high force for ~ 500 cycles. The sample is then stretched until the force reaches the prescribed value and unloaded with the same rate until the force reaches zero. By varying the amplitude of force in each test, the crack extension is measured for different values of G . The crack growth, dc , is measured using an optical microscope. The crack growth is divided by the number of cycles, dn , and plotted against different values of G . The frequency of cyclic loading was fixed for all tests at 0.2 Hz. The resolution of the optical microscope is ~ 5 μ m. We conclude that there is no crack growth after 5000 cycles. The lowest growth per cycle is ~ 5 μ m/5000 = 1 nm cycle $^{-1}$. The energy release rate is calculated as follows:²⁰

$$G = \pi c \sigma^2 / E [1.122 - 0.231(c/b) + 10.55(c/b)^2 - 21.71(c/b)^3 + 30.382(c/b)^4]^2$$

where c is the crack length, b is the width of the specimen, and E is the modulus, and σ is the force divided by the thickness and the width. For PMMA of each mass fraction of a plasticizer, six specimens are tested.

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of interest

There are no conflicts to declare.

Acknowledgements

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